

Tone Detection with a Quadrature Receiver

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1 Introduction

Two-way land mobile radio has long used analog tone signaling for functions such as squelch control and selective calling. Methods of detecting an analog tone of a specified frequency and duration include a diversity of well-known techniques that span the range from mechanical devices to hardware circuits to software solutions. This paper presents a simple technique known as the *quadrature receiver*, which although capable of being implemented in hardware, is usually implemented as software running on a digital signal processor. Simplified approximations of the quadrature receiver can even be implemented as code running on a primitive microprocessor. Regardless of how it is implemented, the quadrature receiver is easy to design, as this paper will shortly demonstrate.

2 Theory of Operation

2.1 Design Objective

The goal of any tone detector is to rapidly identify the presence of a specific tone, even in the presence of noise, while simultaneously avoiding false detection of speech or other signals. The receiver knows only the tone frequency; the amplitude and the phase of the tone are unknown variables. Mathematically, we may designate the tone signal, $x(t)$, as:

$$x(t) = A\cos(\omega t + \phi),$$

where A is the constant amplitude of the tone, ω is the constant angular frequency of the tone and ϕ is an arbitrary constant phase offset term. The receiver must be able to detect the tone without knowledge of A or ϕ .

2.2 Some Trigonometry

The quadrature receiver is based on a few simple trigonometric identities. These are:

$$\cos(a) \cos(b) = \frac{1}{2} [\cos(a + b) + \cos(a - b)],$$

$$\sin(a) \cos(b) = \frac{1}{2} [\sin(a + b) + \sin(a - b)],$$

$$\sin^2(a) + \cos^2(a) = 1.$$

If the receiver can generate $\cos(\omega t)$ and $\sin(\omega t)$, then it can use these trigonometric identities to detect the tone signal, regardless of its phase.

2.3 Receiver Architecture

Figure 1 illustrates the architecture of the quadrature receiver. The receiver generates

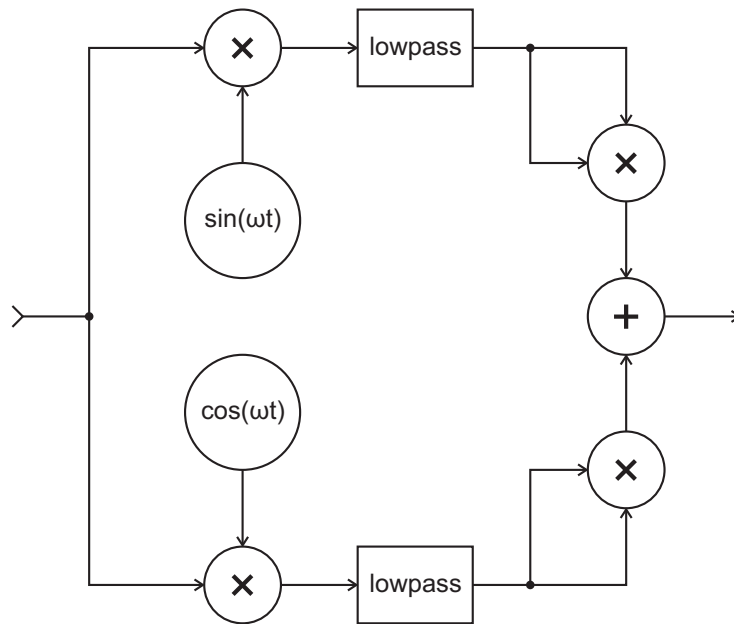


Figure 1: Quadrature Receiver

local oscillator signals, $\cos(\omega t)$ and $\sin(\omega t)$, which it mixes with the received signal. The results are then lowpass filtered, squared and summed. The output of the receiver can then be compared to a threshold to determine the presence or absence of the desired tone signal.

2.4 Mathematical Analysis

Suppose the input signal is: $A\cos(\omega t + \phi)$. Furthermore, suppose the receiver generates: $\cos(\omega t + \theta)$ and $\sin(\omega t + \theta)$. Note that ω is the angular frequency of the tone, ϕ is the arbitrary phase of the received tone, and θ is the arbitrary phase of the receiver local oscillator. The output of the cos mixer is:

$$A\cos(\omega t + \phi) \cdot \cos(\omega t + \theta) = \frac{A}{2} [\cos(2\omega t + \phi + \theta) + \cos(\phi - \theta)],$$

and the output of the sin mixer is:

$$A\cos(\omega t + \phi) \cdot \sin(\omega t + \theta) = \frac{A}{2} [\sin(2\omega t + \phi + \theta) + \sin(\theta - \phi)].$$

For convenience, let the phase difference between the received tone and the local oscillators, $\phi - \theta$, be identified as $\Delta\theta$. If the lowpass filters reject frequencies near 2ω , while passing frequencies near DC, the filtered mixer outputs become:

$$\frac{A}{2} [\cos(\Delta\theta)],$$

and

$$-\frac{A}{2} [\sin(\Delta\theta)],$$

respectively. These signals are then squared and summed, yielding:

$$\frac{A^2}{4} [\cos^2(\Delta\theta) + \sin^2(\Delta\theta)] = \frac{A^2}{4}.$$

Therefore, when the received tone frequency matches the frequency of the local oscillator, the tone detector output is simply DC, regardless of the tone phase.

2.5 Importance of Lowpass Filter

The primary purpose of the lowpass filters thus far has been to remove the high frequency 2ω term from the mixer outputs; however, the lowpass filters also establish the bandwidth and detection time of the quadrature receiver. Suppose the received tone has a frequency of ω_1 , while the tone detector is looking for a tone with a frequency of ω_2 . Let the difference, $\omega_1 - \omega_2$ be defined as $\Delta\omega$, with phase difference $\Delta\theta$, as before. Assuming the filter rejects the 2ω terms, the mixer outputs will then be:

$$\frac{A}{2} [\cos(\Delta\omega t + \Delta\theta)] \text{ and } \frac{A}{2} [\sin(\Delta\omega t + \Delta\theta)].$$

If the lowpass filters pass a frequency of $\Delta\omega$, the tone detector will produce an output. Therefore, to limit the range of frequencies the tone detector will respond to, the lowpass filter must be designed properly.

Suppose the lowpass filter has a cutoff frequency ω_c . This means the tone detector will respond to any frequency in the range $\omega \pm \omega_c$, where ω is the frequency of the local oscillators. The detector response time is inversely proportional to the filter bandwidth, and is approximately $2\pi/\omega_c$.

3 A Specific Example

3.1 Specifications

Suppose we are asked to design a tone detector to the following specifications:

- nominal frequency = 2700 Hz
- response time = 20 msec
- sampling rate = 8000 Hz

3.2 Design

The local oscillator frequency is 2700 Hz. The detector response time determines the filter bandwidth through a reciprocal relationship. A response time of 20 msec requires a filter bandwidth of at least 50 Hz. A simple first order IIR can meet these requirements. Assuming we desire a fixed-point implementation, we may choose the (Q15) filter taps of the IIR to be:

$$b_0 = 369, b_1 = 369, a_0 = 32767, a_1 = -32031.$$

Since the center frequency is 2700 Hz and the sampling rate is 8 KHz, the local oscillator phase increment is 2700 cycles-per-second, divided by 8000 samples-per-second, resulting in 0.3375 cycles-per-sample. Expressed as an unsigned 16-bit fraction, the phase increment becomes 22118.

3.3 Performance

Tests of the quadrature receiver tone detector designed in the previous section yield the following results. Figure 2 shows the frequency response of the tone detector. The detector output has been normalized based on the amplitude of the input signal. This response peaks at 2700 Hz, as expected, and is down to 50 percent at 50 Hz either side of 2700 Hz. Figure 3 shows the time response of the tone detector. The response is at roughly 95 percent of its peak value within 20 msec of the start of the tone. Based on these results, our detector meets its design specifications; however, these tests are not indicative of performance in the real world.

3.4 Effects of Noise

The tests thus far have been in the absence of noise. In a real environment, the tone signal will be corrupted by noise. Figure 4 shows three detector response curves corresponding to different signal-to-noise ratios. The lowest curve represents an SNR of 0 dB, the middle curve represents an SNR of 10 dB, and the upper curve is without any noise, provided as a reference. The plots show that the time response of the detector is not greatly affected by noise, but the detector output level is greatly affected by noise. This must be considered when choosing the detection threshold criteria.

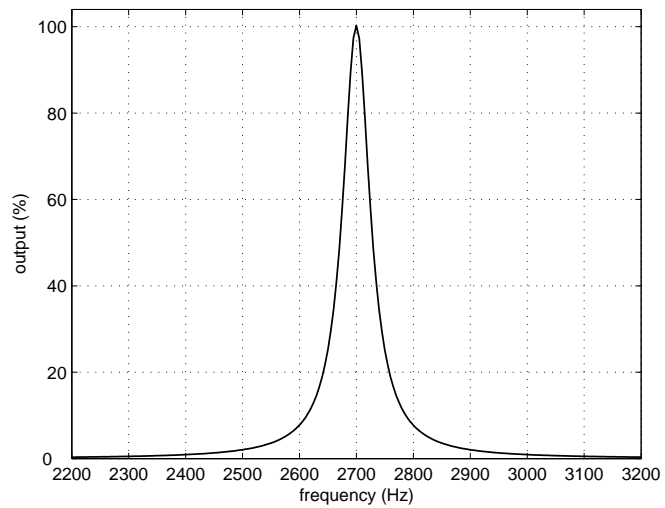


Figure 2: Frequency Response

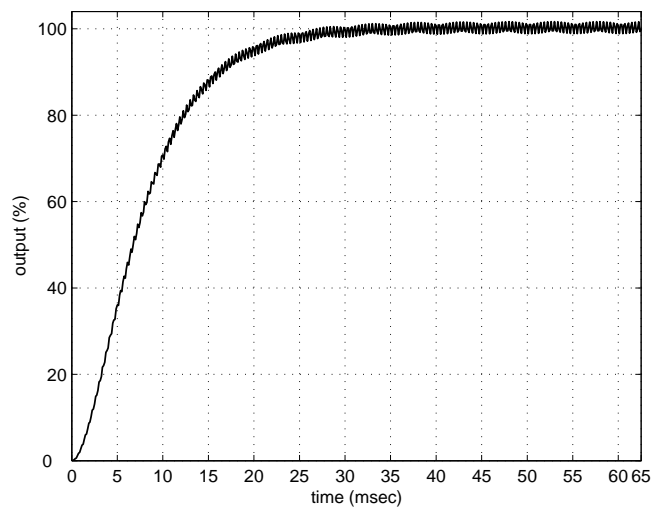


Figure 3: Time Response

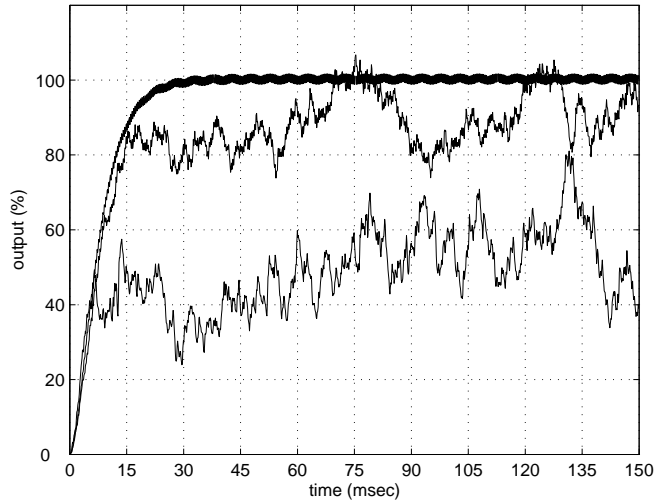


Figure 4: Detection in Noise

3.5 Making It Work

In order for the tone detector to be useful, it must output a logical signal that indicates the presence or absence of the desired tone signal. To do so, the detector should be designed with hysteresis, to prevent the output from chattering in the presence of a noisy signal. Two thresholds must be established: acquisition and loss. The acquisition threshold is the value that must be exceeded to declare the presence of the desired tone. The loss threshold is the value that declares the tone to be lost, once it has been acquired. Both thresholds should be set based on the frequency response of the detector, as shown in Figure 2, and the performance in noise, as shown in Figure 4. For the example here, a reasonable acquisition threshold is 50 percent, and a reasonable loss threshold is 20 percent. Note that the detector should normalize its output based on the average amplitude of the input signal. Failure to properly select the thresholds will result in a detector that either fails to detect a tone in the presence of noise, or creates false detections when the tone is not present.

4 Conclusion

This paper has described the quadrature receiver, a simple and easy-to-understand tone detector that can be implemented in a digital signal processor. The parameters of the detector are easy to adjust, and the performance of the detector is simple to measure. The quadrature receiver described herein is suitable for use in many applications where an analog tone of a specific frequency is used for signaling purposes. With some ingenuity, the quadrature receiver can even be simplified and approxi-

mated to the degree that it may be implemented (with a slight loss in performance) on a simple microprocessor instead of a DSP; however, such modifications are beyond the scope of this paper.

References

- [1] Bernard Sklar, *Digital Communications: Fundamentals and Applications (2nd Edition)*, Prentice Hall, 2001.

